UNPUBLISHED PRELIMINARY DATA

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Gentlemen:

Please accept this letter as a quarterly progress report for the subject contract.

In a previous report ("Rational Approximations to the Response of a Dynamic System Described by a Nonlinear Differential Equation," Final Report, MRI Project No. 2760-P, January 1965), we studied Padé approximations to the solution of the equation

$$y'' + ay + by^2 + cy^3 + d = 0 \ , \quad y(0) = a_o \ , \quad y'(0) = b_o \tag{1}$$

where a , b , c and d are constants. These approximations were extended to account for small damping. The analysis gave valuable insight to the problem of approximating solutions of (1) by continued fractions. For many applications, the procedure lacked sufficient generality since damping had to be small and the forcing function had to be a constant.

In the present study, we generalize (1) by considering the equations

$$(A_o+B_oy)y'' + (C_o+D_oy)y' - 2B_o(y')^2 + E_o + F_oy + G_oy^2 + H_oy^3 = 0 \ , \tag{2}$$

$$y(0) = \alpha_o \ , \quad y'(0) = \beta_o \ , \quad \alpha_o\beta_o \neq 0 \ ,$$

where each of the coefficients is expandable in a Taylor's series about $x = 0$, and $A_o(0) = B_o(0) = C_o(0) = D_o(0) = G_o(0) = H_o(0) = 0$. In particular,

$$A_o = x^2 \sum_{k=0}^{\infty} a_k x^k \quad , \qquad B_o = x^2 \sum_{k=0}^{\infty} b_k x^k \quad ,$$

$$C_o = x \sum_{k=0}^{\infty} c_k x^k \quad , \qquad D_o = x \sum_{k=0}^{\infty} d_k x^k \quad ,$$

$$E_o = \sum_{k=0}^{\infty} e_k x^k \ , \ e_o \neq 0 \ , \qquad F_o = \sum_{k=0}^{\infty} f_k x^k \ , \ f_o \neq 0 \ ,$$

$$G_o = x \sum_{k=0}^{\infty} g_k x^k \quad , \qquad H_o = x \sum_{k=0}^{\infty} h_k x^k \quad . \tag{3}$$

Conditions (3) are not as stringent as they appear in that if the coefficients in (2) do not satisfy (3), application of one or two linear transformations (to be defined below) brings them into the required form. For example, we can obtain approximations to (1) in the presence of damping and an arbitrary forcing function. All we require is that the coefficients in (1) have a Taylor's series expansion about $x = 0$. This is a marked improvement over our previous approximations.

Note that for computational convenience we can approximate the analytic coefficients by polynomials to any desired degree of accuracy over a given interval, compute the approximate solution over this interval, and then use the idea of analytic continuation (see the report mentioned above) to extend the solution to the next adjacent interval. Thus, the solution may be approximated to a high degree of accuracy over any desired range. Let the solution to (2) have a continued fraction expansion of the form

$$y = \cfrac{\alpha_0}{1 + \cfrac{\alpha_1 x}{1 + \cfrac{\alpha_2 x}{1 + \ddots}}} \tag{4}$$

The even approximants of (4) are the main diagonal Padé approximations which have the following properties. Let

$$y_n = \frac{P_n}{Q_n} = \frac{\sum_{k=0}^{n} p_{n,k} x^k}{\sum_{k=0}^{n} q_{n,k} x^k} \tag{5}$$

be the n^{th} order main diagonal Padé approximant. If Q_n is formally divided into P_n, the resulting power series agrees with the power series solution to (2) for the first (2n+1) terms. The polynomials P_n and Q_n both satisfy the relation

$$P_n = \left[1 + (\alpha_{2n-1}+\alpha_{2n})x\right]P_{n-1} - \alpha_{2n-1}\alpha_{2n-2}x^2 P_{n-2} ,$$

$$P_o = \alpha_o , \quad P_1 = \alpha_o(1+\alpha_1 x) , \quad Q_o = 1 \quad \text{and} \quad Q_1 = 1 + (\alpha_1+\alpha_2)x . \tag{6}$$

Thus, rational approximations to the solution of (2) are immediately forthcoming if the values $\alpha_1, \alpha_2, \alpha_3, \ldots$ can be computed. We compute these values by utilizing a linear fractional transformation. Let

$$y = y_o , \quad y_n = \alpha_n(1+xy_{n+1})^{-1} , \quad > n \geq 0 . \tag{7}$$

Repeated application of (7) to (2) and division by $\alpha_n x$ at each step yields

$$(A_{n+1}+B_{n+1}y_{n+1})y''_{n+1} + (C_{n+1}D_{n+1}y_{n+1})y'_{n+1} - 2B_{n+1}(y'_{n+1})^2 + E_{n+1}$$

$$+ F_{n+1}y_{n+1} + G_{n+1}y^2_{n+1} + H_{n+1}y^3_{n+1} = 0 , \tag{8}$$

where

$$A_{n+1} = - A_n - \alpha_n B_n ,$$

$$B_{n+1} = - xA_n ,$$

$$C_{n+1} = - 2x^{-1}(A_n+\alpha_n B_n) - C_n - \alpha_n P_n ,$$

$$D_{n+1} = 2A_n - C_n ,$$

$$E_{n+1} = x^{-1}(\alpha_n^{-1}E_n+F_n+\alpha_n G_n+\alpha_n^2 H_n) ,$$

$$F_{n+1} = - x^{-1}(C_n+\alpha_n D_n) + 3\alpha_n^{-1}E_n + 2F_n + \alpha_n G_n ,$$

$$G_{n+1} = 2x^{-1}A_n - C_n + 3\alpha_n^{-1}xE_n + xF_n ,$$

and

$$H_{n+1} = \alpha_n^{-1}x^2 E_n \quad . \tag{9}$$

It is easily shown that

$$A_{n+1}(0) = B_{n+1}(0) = C_{n+1}(0) = D_{n+1}(0) = G_{n+1}(0) = H_{n+1}(0)$$

and

$$E_{n+1}(0) \neq 0 , \quad F_{n+1}(0) \neq 0 \quad . \tag{10}$$

It follows that after setting $x = 0$ in (8), one gets

$$F_{n+1}(0)\ y_{n+1}(0) = - E_{n+1}(0) \quad ,$$

or

$$\alpha_{n+1} = - \frac{E_{n+1}(0)}{F_{n+1}(0)} \tag{11}$$

and this value is well defined.

Computation of the Padé approximations (5) by using (6) is now easily accomplished.

In the next report, we shall investigate a number of important equations that are special cases of (2) and also we shall present some numerics to illustrate the advantages of these approximations.

Very truly yours,

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(25 copies of report submitted)